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The Eleven-Dimensional Five-Brane

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Abstract

We consider the action, in arbitrary curved background, of the eleven-dimensional five-brane to second order in the curvature of the worldvolume tensor field. We show that this action gives upon double dimensional reduction the action of the Dirichlet four-brane up to the same order. We use this result as a starting point to discuss the structure of the action including terms of higher order in the worldvolume curvature.

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Introduction

The interest in supermembranes and higher-dimensional extended objects was partly triggered by the construction, some years ago, of an action for the eleven-dimensional supermembrane [1]. The bosonic part of this action is given by

$$S = \int d^3\xi [\sqrt{-g} + \epsilon C]. \quad (1)$$

The action contains the eleven scalars (target space coordinates) $X^\mu(\xi^i)$ ($\mu = 0, 1, \dots, 10; i = 0, 1, 2$) and the bosonic fields of eleven-dimensional supergravity which act as background fields⁴. These are the metric g and the three-form C :

$$g_{ij} = \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}, \quad C_{ijk} = \partial_i X^\mu \partial_j X^\nu \partial_k X^\rho C_{\mu\nu\rho}. \quad (2)$$

The action (1) is invariant under the eleven-dimensional gauge transformations

$$\delta C = \partial\chi. \quad (3)$$

A particularly interesting property of this action is that the requirement of the so-called kappa symmetry of the full action (i.e. including the fermions) leads to the equations of motion of eleven-dimensional supergravity [1].

It has by now become clear that the other basic extended object in eleven dimensions, besides the membrane, is a five-brane. The role of the five-brane has been discussed recently in [3, 4, 5]. Given the interest in the supermembrane action, it is clearly of interest to construct an action, similar to (1), for the eleven-dimensional super five-brane. Partial results in this direction have already been obtained in the literature. For instance, it is known that the (bosonic part of) the action contains, besides the usual embedding coordinates $X^\mu(\xi^i)$ ($i = 0, 1, \dots, 5$), a self-dual worldvolume two-form W_{ij} [6, 7]. The action consists of two parts, a kinetic term and a Wess-Zumino term. The kinetic term is non-linear in the curvature of the two-form. The expression of this part to lowest order, quadratic, terms in the curvature has been given in [8]. The Wess-Zumino term is of finite order in the two-form

⁴ For the background fields we use the notation and conventions of Ref. [2].

curvature and has been given in [9]⁵. It is the aim of this letter to investigate further the structure of the five-brane action, in particular its relation with D -branes [10] and more specifically with the ten-dimensional Dirichlet four-brane action.

1 The five-brane action at quadratic order

We first consider the five-brane action at quadratic order in the curvature of the worldvolume gauge field in a curved background. Before doing this, it is instructive to first consider the background fields $\{e_\mu{}^a, C_{\mu\nu\rho}\}$ that describe eleven-dimensional supergravity. In particular the equation of motion for C is given by

$$\partial \left({}^* \partial C + \frac{105}{4} C \partial C \right) = 0. \quad (4)$$

This implies that there exists a dual six-form \tilde{C} [3, 9] with⁶

$$G(\tilde{C}) = {}^* G(C), \quad (5)$$

where

$$G(C) = \partial C, \quad G(\tilde{C}) = \partial \tilde{C} - \frac{105}{4} C \partial C. \quad (6)$$

Note that \tilde{C} transforms under the gauge transformation (3) as

$$\delta \tilde{C} = -\frac{105}{4} \partial \chi C, \quad (7)$$

⁵We thank Paul Townsend for bringing this reference to our attention.

⁶This relation defines implicitly \tilde{C} in terms of C . Given $\partial \tilde{C}$, \tilde{C} itself can be obtained, for instance, from

$$\tilde{C}_{\mu_1 \dots \mu_6} = 7 \int_0^1 d\lambda \lambda^6 \partial_{[\mu_1} \tilde{C}_{\mu_2 \dots \mu_6 \nu]}(\lambda x) x^\nu.$$

For a general k -form potential one has

$$A_{\mu_1 \dots \mu_k} = (-1)^k (k+1) \int_0^1 d\lambda \lambda^k \partial_{[\mu_1} A_{\mu_2 \dots \mu_{k+1}]}(\lambda x) x^{\mu_{k+1}}.$$

so that C and \tilde{C} occur at the same time⁷. It turns out that C also occurs inside the curvature \mathcal{H} of the worldvolume two-form W as a Chern-Simons term [8]:

$$\mathcal{H} = 3(\partial W - \tfrac{1}{2}C) . \quad (8)$$

The gauge invariance of the field strength \mathcal{H} implies the following gauge transformation law for W :

$$\delta W = \tfrac{1}{2}\chi . \quad (9)$$

Combining the results of [6, 7, 8, 9] the five-brane action to second order in \mathcal{H} is given by

$$S = \int d^6\xi \left[\sqrt{-g} \left(1 + \tfrac{1}{2}\mathcal{H}^2 \right) + \epsilon \left(\tfrac{1}{70}\tilde{C} + \tfrac{3}{4}\partial W C \right) \right] . \quad (10)$$

Following [2, 12] this action is related to the equations of motion of the five-brane coordinates $\{X^\mu, W_{ij}\}$ as follows. First one writes down the usual equations of motion following from (10). Next, one substitutes in these equations the following (lowest order) selfduality condition:

$$\mathcal{H} = *\mathcal{H} . \quad (11)$$

This can be done consistently everywhere provided that the action (10) is invariant under Poincaré duality. By this we mean that if we dualize the two-form W into a dual two-form \tilde{W} , the action as a function of W is identical to the action as a function of \tilde{W} . To Poincaré-dualize the above action we consider \mathcal{H} as the fundamental field (instead of W) and add the following term to the action

$$\int d^6\xi \tfrac{1}{2}\epsilon\partial\tilde{W} \left(\mathcal{H} + \tfrac{3}{2}C \right) , \quad (12)$$

where \tilde{W}_{ij} is a Lagrange multiplier that enforces the Bianchi identity for \mathcal{H} . We then eliminate \mathcal{H} using its equation of motion

$$*\mathcal{H} = \tilde{\mathcal{H}} \equiv 3(\partial\tilde{W} - \tfrac{1}{2}C) . \quad (13)$$

⁷This is similar to what happens in the construction of certain extensions of the eleven-dimensional Poincaré superalgebra [11].

One may verify that this procedure indeed leads to the required self-duality of the five-brane action (to quadratic order).

We finally note that all terms in the action are fixed by gauge invariance and selfduality [9]. For instance, the relative factor between the two terms in the Wess-Zumino term in (10) are fixed by gauge invariance while the coefficient of the second term in the Wess-Zumino term is related to the Chern-Simons term inside the \mathcal{H} curvature of the kinetic term via Poincaré selfduality.

2 Double Dimensional Reduction

In this section we put hats on top of all the six- or eleven-dimensional objects to distinguish them from the five- and ten-dimensional ones. In the target-space dimensional reduction we use the results of [2] where this reduction from eleven-dimensional supergravity to the ten-dimensional type IIA supergravity was performed. For the accompanying world-volume dimensional reduction we make the standard ansatz:

$$\{\hat{\xi}^i\} = \{\xi^i, \rho\}, \quad \{\hat{X}^{\hat{\mu}}(\hat{\xi}^i)\} = \{X^\mu(\xi^i), \hat{X}^{10} = \rho\}, \quad (14)$$

which implies

$$\partial_i \hat{X}^\mu = \partial_i X^\mu, \quad \partial_\rho \hat{X}^{10} = 1. \quad (15)$$

Now, using the Kaluza-Klein ansatz for the eleven-dimensional target-space metric of Ref. [2], we get for the induced world-volume metric

$$\left\{ \begin{array}{l} \hat{g}_{ij} = e^{-\frac{2}{3}\phi} g_{ij} - e^{\frac{4}{3}\phi} A_i^{(1)} A_j^{(1)}, \\ \hat{g}_{i\rho} = -e^{\frac{4}{3}\phi} A_i^{(1)}, \\ \hat{g}_{\rho\rho} = -e^{\frac{4}{3}\phi}, \end{array} \right. \quad (16)$$

where $A_i^{(1)}$ is the pull-back of the ten-dimensional vector field $A_\mu^{(1)}$ etc. Using this result and

$$\hat{W}_{ij} = W_{ij}, \quad \hat{W}_{i\rho} = V_i, \quad (17)$$

one gets for the kinetic term

$$\sqrt{-\hat{g}} \left[1 + \frac{1}{2} \hat{\mathcal{H}}^2 \right] = \sqrt{g} \left[e^{-\phi} + \frac{1}{2} e^{\phi} \mathcal{H}^2 - \frac{3}{2} e^{-\phi} \mathcal{F}^2 \right] , \quad (18)$$

where the three- and two-form field strengths \mathcal{H} and \mathcal{F} are given by

$$\mathcal{H} = 3 \left(\partial W - \frac{1}{2} C - A^{(1)} \mathcal{F} \right) , \quad (19)$$

$$\mathcal{F} = 2\partial V - B^{(1)} . \quad (20)$$

Next, we reduce the Wess-Zumino term. With the above definitions and results one gets

$$\frac{3}{4} \hat{\epsilon} \partial \hat{W} \hat{C} = \frac{3}{2} \epsilon \left(\partial W B^{(1)} - \partial V C \right) , \quad (21)$$

We define

$$\hat{\tilde{C}}_{\mu_1 \dots \mu_5 \underline{\rho}} = \frac{7}{6} \tilde{C}_{\mu_1 \dots \mu_5} , \quad (22)$$

so that

$$G(\tilde{C}) = - {}^* G(C) , \quad (23)$$

with

$$G(C) = \partial C - 2\partial B^{(1)} A^{(1)} , \quad G(\tilde{C}) = \partial \tilde{C} - \frac{15}{2} \left(C \partial B^{(1)} + B^{(1)} \partial C \right) . \quad (24)$$

These equations follow from dimensionally reducing the defining equations (5,6) for $\hat{\tilde{C}}$.

Summarizing our results, we get the following five-dimensional action up to quadratic order:

$$\begin{aligned} S = & \int d^5 \xi \sqrt{g} \left(e^{-\phi} + \frac{1}{2} e^{\phi} \mathcal{H}^2 - \frac{3}{2} e^{-\phi} \mathcal{F}^2 \right) \\ & + \epsilon \left[\frac{1}{10} \tilde{C} + \frac{3}{2} (\partial W B^{(1)} - \partial V C) \right] . \end{aligned} \quad (25)$$

This action is invariant under the following gauge transformations:

$$\begin{aligned}
\delta C &= \partial\chi + 2B^{(1)}\partial\Lambda^{(1)}, & \delta W &= \frac{1}{2}\chi - 2\partial\Lambda^{(1)}V, \\
\delta B^{(1)} &= \partial\eta^{(1)}, & \delta V &= \frac{1}{2}\eta^{(1)}, \\
\delta A^{(1)} &= \partial\Lambda^{(1)}, & \delta\tilde{C} &= -\frac{15}{2}(\partial\chi B^{(1)} - \partial\eta^{(1)}C).
\end{aligned} \tag{26}$$

By construction it is also selfdual in the sense that the action remains the same in form if one dualizes the vector V to a tensor \tilde{W} and the tensor W to a vector \tilde{V} . This means that it is consistent to substitute the following relation

$$\mathcal{H} = e^{-\phi} \star \mathcal{F}, \tag{27}$$

which is the double dimensional reduction of the selfduality condition (11) into the field equations. The consistency is evident if one compares the equations of motion and Bianchi identities of W and V :⁸

$$\left\{ \begin{array}{ll} \nabla_i(e^\phi \mathcal{H}^{ijk}) + 3 \star H^{(1)jk} & = 0, \\ \nabla_i(e^{-\phi} \mathcal{F}^{ij}) - 6 \star [G(C) + e^\phi F^{(1)} \star \mathcal{H}]^j & = 0, \\ \nabla_i \star \mathcal{H}^{ij} - 6 \star [G(C) + F^{(1)} \mathcal{F}]^j & = 0, \\ \nabla_i \star \mathcal{F}^{ijk} + 3 \star H^{(1)jk} & = 0. \end{array} \right. \tag{28}$$

Then, we can use relation (27) to consistently eliminate W from the field equations of V and X^μ . The equations corresponding to X^μ are given by⁹:

⁸ Gauge invariance guarantees that the field equations can be given in terms of curvatures only. In order to achieve this in the equation of motion of V one has to substitute the equation of motion of W .

⁹We have substituted the equations of motion of both V and W to simplify the expressions.

$$\begin{aligned}
& 2g_{\mu\sigma}\nabla_i(T^{ij}\partial_j X^\sigma) - 9e^{-\phi}\mathcal{F}^{ij}H_{ij\mu}^{(1)} \\
& + (K - e^\phi\mathcal{H}^2)\partial_\mu\phi - 6e^\phi\mathcal{H}^{ijk}\left[G(C)_{ijk\mu} + F_{ij}^{(1)}\mathcal{F}_{k\mu}\right] \\
& + \frac{1}{\sqrt{g}}\epsilon^{ijklm}\left[\frac{3}{5}G(\tilde{C})_{ijklm\mu} - 3\mathcal{F}_{ij}G(C)_{klm\mu} - \frac{3}{2}\mathcal{H}_{ijk}H_{lm\mu}^{(1)} + \frac{3}{2}F_{ij}^{(1)}\mathcal{F}_{kl}\mathcal{F}_{m\mu}\right] = 0,
\end{aligned} \tag{29}$$

where T_{ij} is the energy-momentum tensor whose explicit form is not necessary here and K is the kinetic term in the action. We next eliminate \mathcal{H} using the relation (27) from the equation of motion of V and X^μ and obtain the following equations respectively:

$$\nabla_i(e^{-\phi}\mathcal{F}^{ij}) - 6\star[G(C) + F^{(1)}\mathcal{F}]^j = 0, \tag{30}$$

$$\begin{aligned}
& 2g_{\mu\sigma}\nabla_i(T'^{ij}\partial_j X^\sigma) - 18e^{-\phi}\mathcal{F}^{ij}H_{ij\mu}^{(1)} + K'\partial_\mu\phi \\
& + \frac{1}{\sqrt{g}}\epsilon^{ijklm}\left[\frac{3}{5}G(\tilde{C})_{ijklm\mu} - 6\mathcal{F}_{ij}G(C)_{klm\mu} - \frac{3}{2}F_{ij}^{(1)}\mathcal{F}_{kl}\mathcal{F}_{m\mu}\right] = 0,
\end{aligned} \tag{31}$$

Here T'^{ij} follows from T^{ij} by eliminating \mathcal{H} and is the energy momentum tensor corresponding to a kinetic term K' where the \mathcal{H}^2 term has been removed and where the factor in front of the \mathcal{F}^2 term has been doubled.

We find that the field equations (30,31) follow from the following action

$$\begin{aligned}
S &= \int d^5\xi \sqrt{g}e^{-\phi}\left(1 - 3\mathcal{F}^2\right) \\
&+ \epsilon\left[\frac{1}{10}\tilde{C} - 3\partial VC + \frac{3}{4}B^{(1)}C - \frac{3}{2}A^{(1)}\mathcal{F}\mathcal{F}\right].
\end{aligned} \tag{32}$$

This is exactly the Dirichlet four-brane action, up to quadratic order in the kinetic term, constructed in [13]. Note that the result for the Wess-Zumino term is complete. We conclude that the five-brane action in a curved eleven-dimensional background leads, upon double dimensional reduction, to the Dirichlet four-brane action in a curved ten-dimensional Type IIA background. This extends the analysis of [8] to nonzero Ramond-Ramond background fields.

Finally, we note that it is not too difficult to find the massive, i.e. $m \neq 0$, extension of the ten-dimensional Dirichlet four-brane action [13]. The only changes are that in the defining relations (23–24), one has to use the massive curvatures

$$G(C)_m = G(C)_{m=0} + \frac{m}{2}(B^{(1)})^2, \quad G(\tilde{C})_m = G(\tilde{C})_{m=0} - \frac{15m}{6}(B^{(1)})^3, \quad (33)$$

and, furthermore, one has to add a topological term for V to the four-brane action:

$$S_m = S_{m=0} - 2m \int d^5\xi \, \epsilon V \partial V \partial V. \quad (34)$$

The eleven-dimensional origin of these mass-dependent terms is an open issue [15, 13].

3 Nonlinearities

We will briefly discuss our efforts to extend the quadratic kinetic term (10) of the eleven-dimensional five-brane to higher orders in the worldvolume curvature \mathcal{H} . In the present context, the action is constrained by two conditions: it should be self-dual in $d = 6$, in the sense discussed in Section 1, and, on double dimensional reduction to $d = 5$ ($d = 10$), it must give the Dirichlet four-brane action to all orders in \mathcal{F} .

Now, the kinetic term in the $d = 10$ four-brane action, to all orders in \mathcal{F} , is known. It is of the Born-Infeld type, explicitly¹⁰:

$$S_{\text{kin}}^{(4)} = \int d^5\xi e^{-\phi} \sqrt{\det(g_{ij} + \mathcal{F}_{ij})}, \quad (35)$$

with \mathcal{F} as in (20). This action is related by T -duality to the kinetic terms for the other $d = 10$ D-branes [15, 16, 17].

The determinant in (35) is of fourth order in \mathcal{F} , i.e.

$$S_{\text{kin}}^{(4)} = \int d^5\xi e^{-\phi} \sqrt{g} \left\{ 1 + \frac{1}{2}\mathcal{F}^2 + \frac{1}{8}(\mathcal{F}^2)^2 - \frac{1}{4}\mathcal{F}^4 \right\}^{1/2}, \quad (36)$$

¹⁰ This action has the property that after dualizing the worldvolume vector into a tensor the dual action is of the same Born-Infeld form with \mathcal{F} replaced by $i^*\mathcal{H}$. The same is true for all worldvolume dimensions $d \leq 5$ but not for $d \geq 6$ [14].

with

$$\mathcal{F}^2 \equiv \mathcal{F}^{ij} \mathcal{F}_{ij}, \quad \mathcal{F}^4 \equiv \mathcal{F}^{ij} \mathcal{F}_{jk} \mathcal{F}^{kl} \mathcal{F}_{li}. \quad (37)$$

It is therefore tempting to start with an action in $d = 6$ which is also of fourth order in \mathcal{H} under a square root. However, computer calculations we performed show that for such a fourth order action with a square root to be self-dual, the argument of the root must be either of second order in \mathcal{H} , or it must be a full square, giving rise to the kinetic term in (10). Neither possibility reduces to (35) in $d = 5$ (after eliminating \mathcal{H} using the nonlinear¹¹ selfduality constraint). Extensions of the argument under the square root to higher orders in \mathcal{H} which vanish on double dimensional reduction are possible. Due to the complexity of the calculations we have not been able to check the selfduality in this case.

Another approach is to extend more systematically the lowest order kinetic term in (10) to higher orders in \mathcal{H} . At fourth order, the most general action that we can write is

$$\begin{aligned} S = & \int d^6 \xi \left\{ \sqrt{-g} \left[1 + \frac{1}{2} \mathcal{H}^2 + a(\mathcal{H}^2)^2 + b(\mathcal{H}^{2ij})^2 + c\mathcal{H}^4 + \mathcal{O}(\mathcal{H}^6) \right] \right. \\ & \left. + \epsilon \left(\frac{1}{70} \tilde{C} + \frac{3}{4} \partial W C \right) \right\}, \end{aligned} \quad (38)$$

where we have used the following notation:

$$\begin{aligned} \mathcal{H}^2 & \equiv \mathcal{H}^{ijk} \mathcal{H}_{ijk}, & \mathcal{H}^{2ij} & \equiv \mathcal{H}^{imn} \mathcal{H}^j_{mn}, \\ (\mathcal{H}^{2ij})^2 & \equiv \mathcal{H}^{2ij} \mathcal{H}^2_{ij}, & \mathcal{H}^4 & \equiv \mathcal{H}^{ijk} \mathcal{H}_{ilm} \mathcal{H}^{ln}_j \mathcal{H}^m_{kn}, \end{aligned} \quad (39)$$

for all the possible terms that can appear to order four in \mathcal{H} . It turns out that, to order \mathcal{H}^4 , Poincaré self-duality does not impose any constraint on the coefficients a, b, c . This is related to the fact that all three \mathcal{H}^4 invariants in (38) have the property that they are invariant under the replacement of \mathcal{H} by ${}^* \mathcal{H}$. We expect that an expansion of the type (38) will put some constraints on the expansion coefficients but not determine them uniquely. Most likely

¹¹Note that the selfduality constraint $\mathcal{H} = {}^* \mathcal{H}$ gets modified with terms of higher order in \mathcal{H} .

these constraints will be consistent with the Born-Infeld action upon double dimensional reduction. In view of this we have not pursued this approach further.

A special property of (35) opens another possibility. Note that

$$[\det(g_{ij} + \mathcal{F}_{ij})]^2 = g \det(g_{ij} - \mathcal{F}_{ij}^2). \quad (40)$$

The right hand side of (40) can be generalised to $d = 6$: $\det(g_{ij} - \mathcal{H}_{ij}^2)$. However, an action involving this determinant does not appear to be self-dual, nor does it give the correct Dirichlet four-brane action on reduction.

Finally, we note that an alternative approach to investigate the nonlinearities is to introduce an independent worldvolume metric. Solving for its equation of motion automatically introduces nonlinearities. This approach is known to work in relating the eleven-dimensional membrane to the ten-dimensional Dirichlet two-brane via direct dimensional reduction [8]. A conjecture for the five-brane action has been given in the same reference. Note that this conjecture satisfies our criterion of Poincaré selfduality¹², although it is not unique in this sense. The double dimensional reduction of the metric leads to an independent worldvolume metric, vector and scalar. It would be interesting to see whether eliminating these fields plus using the (reduced) selfduality condition reproduces the Dirichlet four-brane action.

4 Conclusion

The problem of finding the complete nonlinear kinetic term of the eleven-dimensional five-brane action remains. Given the fact that this kinetic term must be related to the nonlinear Born-Infeld term upon double dimensional reduction, we expect that an answer to all orders must exist. One would also expect the answer to be simple and to have some geometrical interpretation, like the Born-Infeld action. Finding this complete nonlinear action is a challenge. A first step forwards involves finding out which restrictions the Poincaré selfduality imposes on the form of the action. This problem is similar to finding nonlinear extensions of electrodynamics that are consistent with electric-magnetic duality, the only difference being that in our

¹²The corresponding (linear in \mathcal{H}) selfduality condition contains the independent worldvolume metric and not the induced metric. Eliminating the worldvolume metric leads to a nonlinear selfduality condition.

case we are dealing with a tensor instead of a Maxwell vector field. In the case of electrodynamics the selfdual Lagrangians are related to solutions of certain Hamilton-Jacobi equations [18]. It would be interesting to extend the programme of [18] to the case of an antisymmetric tensor field.

Finally, we have restricted the discussion to the bosonic part of the action only. The full super five-brane action, including the fermions, must be kappa-symmetric in order to have the correct Bose-Fermi matching. In the case of the supermembrane the kappa-symmetric extension is known [1]. It involves replacing all bosonic fields by superfields. In this context, it is of interest to note that the five-brane action contains the dual six-form field \tilde{C} . This suggests that supersymmetrizing the five-brane action requires a superspace description of eleven-dimensional supergravity that involves a six-form superfield $\tilde{C}(X, \theta)$. It would be interesting to find this superspace formulation.

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